REDSPOT

IGCSE

(Syllabus 0580)

MATHEMATICS

Paper 4 (Extended) - All Variants

(Topical)

period	**2018 to June-2024**
contents	**June & November, Paper 4 (P41, P42 & P43) Worked Solutions**
form	**Topic By Topic**
compiled for	**IGCSE**

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TOPIC 2

Ratio & Proportion, Rates, Time

1. Brad travelled from his home in New York to Chamonix.

- He left his home at 16 30 and travelled by taxi to the airport in New York.
  This journey took 55 minutes and had an average speed of 18 km/h.
- He then travelled by plane to Geneva, departing from New York at 22 15.
  The flight path can be taken as an arc of a circle of radius 6400 km with a sector angle of 55.5°.
  The local time in Geneva is 6 hours ahead of the local time in New York.
  Brad arrived in Geneva at 11 25 the next day.
- To complete his journey, Brad travelled by bus from Geneva to Chamonix.
  This journey started at 13 00 and took 1 hour 36 minutes.
  The average speed was 65 km/h.
  The local time in Chamonix is the same as the local time in Geneva.

Find the overall average speed of Brad's journey from his home in New York to Chamonix.
Show all your working and give your answer in km/h.

.... km/h [11]

[June/2019/P41/Q11]

2. Here is part of a train timetable for a journey from London to Marseille.
All times given are in local time.
The local time in Marseille is 1 hour ahead of the local time in London.

London	07 19
Ashford	07 55
Lyon	13 00
Avignon	14 08
Marseille	14 46

(i) Work out the total journey time from London to Marseille.
Give your answer in hours and minutes.

.... h .... min [2]

(ii) The distance from London to Ashford is 90 km.
The local time in London is the same as the local time in Ashford.

Work out the average speed, in km/h, of the train between London and Ashford.

.... km/h [3]

(iii) During the journey, the train takes 35 seconds to completely cross a bridge.
The average speed of the train during this crossing is 90 km/h.
The length of the train is 95 metres.

Calculate the length, in metres, of this bridge.

.... m [4]

[June/2019/P43/Q1(a)]

3. Ali and Mo share a sum of money in the ratio Ali : Mo = 9 : 7.
Ali receives \$600 more than Mo.

Calculate how much each receives.

Ali \$ ....

Mo \$ ....

[3]

[Nov/2019/P41/Q2(a)]

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4. (a) Mohsin has 600 pear trees and 720 apple trees on his farm.

(i) Write the ratio pear trees : apple trees in its simplest form.

.... : .... [1]

(ii) Each apple tree produces 16 boxes of apples each year.
One box contains 18 kg of apples.

Calculate the total mass of apples produced by the 720 trees in one year.
Give your answer in standard form.

.... kg [3]

(b) (i) One week, the total mass of pears picked was 18 540 kg.
For this week, the ratio mass of apples : mass of pears = 13 : 9.

Find the mass of apples picked that week.

.... kg [2]

(ii) The apples cost Mohsin \$0.85 per kilogram to produce.
He sells them at a profit of 60%.

Work out the selling price per kilogram of the apples.

\$ .... [2]

[Nov/2019/P42/Q1(a,b)]

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5. Car A and car B take part in a race around a circular track.
One lap of the track measures 7.6 km.

Car A takes 2 minutes and 40 seconds to complete each lap of the track.
Car B takes 2 minutes and 25 seconds to complete each lap of the track.
Both cars travel at a constant speed.

(a) Calculate the speed of car A.
Give your answer in kilometres per hour.

.... km/h [3]

(b) Both cars start the race from the same position, S, at the same time.

(i) Find the time taken when both car A and car B are next at position S **at the same time**.
Give your answer in minutes and seconds.

.... min .... s [4]

(ii) Find the distance that car A has travelled at this time.

.... km [2]

[Nov/2019/P42/Q9]

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6. Asif cycles a distance of 105 km.

On the first part of his journey he cycles 60 km in 2 hours 24 minutes.

On the second part of his journey he cycles 45 km at 20 km/h.

Find his average speed for the whole journey.

.... km/h [4]

[Nov/2019/P43/Q1(d)]

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7. Arjun and Gretal each pay rent.

In 2018, the ratio of the amount each paid in rent was Arjun : Gretal = 5 : 7.

In 2019, the ratio of the amount each paid in rent was Arjun : Gretal = 9 : 13.

Arjun paid the same amount of rent in both 2018 and 2019.

Gretal paid $290 more rent in 2019 than she did in 2018.

Work out the amount Arjun paid in rent in 2019.

$ .... [4]

[June/2020/P41/Q1(e)]

ANSWERS

Topic 2 - Ratio & Proportion, Rate, Time

1.. From Home to New York Airport.

Journey time $= \frac{55}{60}$ h

Distance travelled = speed × time

$$= 18 \times \frac{55}{60} = 16.5 \text{ km}$$

From New York to Geneva.

Distance travelled = arc length of the sector

$$= \frac{55.5°}{360°} \times 2\pi(6400) \approx 6199 \text{ km}$$

From Geneva to Chamonix.

Journey time = 1 h 36 min = 1.6 hours.

Distance travelled $= 65 \times 1.6 = 104$ km

Now, Total distance travelled for the whole journey $= 16.5 + 6199 + 104 = 6319.5$ km

Brad starts his journey from home at 16 30

His arrival time in Chamonix = 13 00 + 01 36
= 14 36 next day

Arrival time in New York local time
= 14 36 − 06 00 = 08 36 next day

∴ Total time for the whole journey
= from 16 30 till 08 36 next day
= 16 h 6 min = 16.1 h

$$\text{Overall average speed} = \frac{\text{total distance}}{\text{total time}}$$

$$= \frac{6319.5}{16.1} = 392.5$$

$$\approx 393 \text{ km/h}$$

2. **(i)** Departure time from London = 07 19

Arrival time in Marseille as per London local time = 13 46

Journey time from London to Marseille
= 13 46 − 07 19 = 06 27
= 6 hours 27 minutes.

(ii) Time taken = 07 55 − 07 19

$$= 36 \text{ minutes} = \frac{36}{60} \text{ h} = 0.6 \text{ h}$$

$$\therefore \text{ Average speed} = \frac{90}{0.6} = 150 \text{ km/h}$$

(iii) Av. speed of train = 90 km/h

$$= 90 \times \frac{1000}{3600} = 25 \text{ m/s}$$

Distance covered by train to completely cross the bridge $= 25 \times 35 = 875$ m

This distance includes the length of bridge and the length of train.

Thus, length of bridge = 875 − 95 = 780 m.

3. Let Mo receives $\$x$,

then Ali receives $= \$(x + 600)$

Given that, $\frac{x+600}{x} = \frac{9}{7}$

$\Rightarrow 7x + 4200 = 9x$

$\Rightarrow 2x = 4200 \Rightarrow x = 2100$

∴ Ali received, \$2100 + \$600 = \$2700

Mo received, \$2100

4. **(a)** **(i)**

Pear trees	:	Apple trees
600	:	720
5	:	6

(ii) One tree produce apples $= 16 \times 18$
= 288 kg

720 trees produce apples $= 288 \times 720$
= 207 360 kg.
$= 2.0736 \times 10^5$ kg

(b) **(i)** 9 parts —— 18540 kg

13 parts —— $\frac{18540}{9} \times 13 = 26780$ kg

∴ Mass of apples picked = 26 780 kg

(ii) Selling price per kg $= \$0.85 + \frac{60}{100} \times \$0.85$
$= \$0.85 + \$0.51$
$= \$1.36$

5. **(a)** 2 minutes 40 seconds = 160 s

$$= \frac{160}{3600} \text{ h} = \frac{2}{45} \text{ h}$$

$$\therefore \text{ Speed of } A = \frac{7.6}{\frac{2}{45}}$$

$$= 7.6 \times \frac{45}{2} = 171 \text{ km/h}$$

(b) (i) Time to complete one lap,

for car $A = 160$ seconds.

for car $B = (2 \times 60) + 25 = 145$ seconds.

Prime factors of $160 = 2^5 \times 5$

Prime factors of $145 = 5 \times 29$

L.C.M. of 160 and $145 = 2^5 \times 5 \times 29$

$= 4640$

Thus, after 4640 seconds, both cars A and B will be again at position S

$$4640 \text{ s} = \frac{4640}{60} \text{ minutes}$$
$$= 77\tfrac{1}{3} \text{ minutes}$$
$$= 77 \text{ minutes } 20 \text{ seconds.}$$

(ii) Time taken by car $A = \frac{4640}{3600} = \frac{58}{45}$ h

Distance covered = speed × time

$$= 171 \times \frac{58}{45} = 220.4 \text{ km.}$$

6. Time taken for the 1st part = 2 h 24 minutes

$= 2.4$ h

Time taken for the 2nd part $= \frac{45}{20}$ h $= 2.25$ h

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}}$$
$$= \frac{105}{2.4 + 2.25}$$
$$= \frac{105}{4.65} \approx 22.6 \text{ km/h}$$

7. In 2018, Let the rent paid by Arjun and Gretal be $\$5x$ and $\$7x$ respectively

In 2019, rent paid by Arjun $= \$5x$

And rent paid by Gretal $= \$7x + \$290$

Given that, in 2019, the ratio of rent paid $= 9 : 13$

$\Rightarrow \frac{5x}{7x + 290} = \frac{9}{13}$

$\Rightarrow 13(5x) = 9(7x + 290)$

$\Rightarrow 65x = 63x + 2610$

$\Rightarrow 2x = 2610 \quad \Rightarrow \quad x = 1305$

$\therefore$ Rent paid by Arjun in 2019 $= \$5x$

$= \$5(1305)$

$= \$6525$

Alternative Method:.

In 2018, Let Arjun paid rent $= \$5x$

Let Gretal paid rent $= \$7x$

In 2019, Let Arjun paid rent $= \$9y$

Let Gretal paid rent $= \$13y$

Arjun paid the same rent in both years,

$\therefore \quad 5x = 9y \quad \Rightarrow \quad x = \frac{9}{5}y$ .... (1)

Gretal paid \$290 more in 2019 than in 2018,

$\therefore \quad 13y - 7x = 290$ .... (2)

Substitute eq. (1) into eq. (2)

$13y - 7\left(\frac{9}{5}y\right) = 290$

$\Rightarrow \frac{65y - 63y}{5} = 290$

$\Rightarrow 2y = 290(5) \quad \Rightarrow \quad y = 725$

$\therefore$ Rent paid by Arjun in 2019 $= \$9y$

$= \$9(725)$

$= \$6525$

8. (i) Sum of ratio $= 7 + 5 = 12$

$\therefore \quad \frac{7}{12} \times \$24 = \$14, \quad \frac{5}{12} \times \$24 = \$10$

(ii) $\frac{\$24.60}{\$2870} = \frac{3}{350}$

9. (a) Total fees $= (54 \times \$15) + (18 \times \$25)$

$= \$810 + \$450 = \$1260$

(b) $100\% - 4.5\% = 95.5\%$

95.5% of total income $= \$37054$

$\frac{95.5}{100} \times$ total income $= \$37054$

total income $= \$37054 \times \frac{100}{95.5} = \$38800$

(c) (i) men : women : children

5 : 4

↘ ↑ ↘

3 : 7

15 : 12 : 28

$\therefore$ men : women : children $= 15 : 12 : 28$

(ii) Ratio factor $= \frac{224}{28} = 8$

men : women : children

15 : 12 : 28

multiply by 8

120 : 96 : 224

$\therefore$ Total number of men and women

$= 120 + 96 = 216$

TOPIC 18

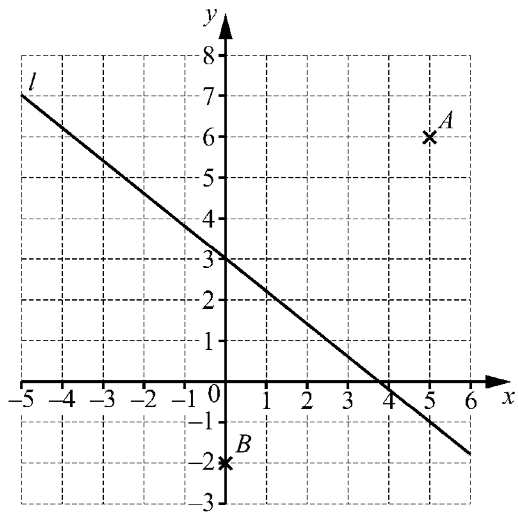
Coordinate Geometry

1. Find the equation of the straight line that is perpendicular to the line $y = \frac{1}{2}x + 1$ and passes through the point (1, 3).

.... [3]

[June/2018/P43/Q9(a)]

2.



(a) Write down the co-ordinates of A.

(.................... , ...) [1]

(b) Find the equation of line l in the form $y = mx + c$.

y = ... [3]

(c) Write down the equation of the line parallel to line l that passes through the point B.

.... [2]

(d) C is the point (8, 14).

(i) Write down the equation of the line perpendicular to line l that passes through the point C.

.... [3]

(ii) Calculate the length of AC.

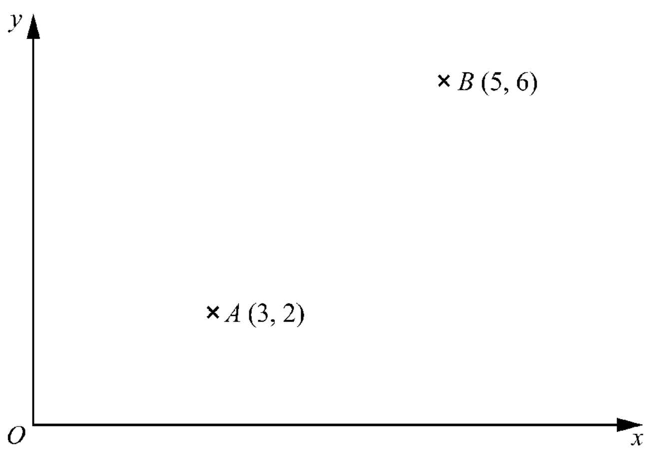
.... [3]

(iii) Find the co-ordinates of the mid-point of BC.

(.................... , ...) [2]

[Nov/2018/P41/Q8]

3.



(i) Find the column vector $\overrightarrow{AB}$.

$\overrightarrow{AB} = \begin{pmatrix} \\ \\ \end{pmatrix}$ [1]

(ii) Find $|\overrightarrow{AB}|$.

$|\overrightarrow{AB}|$ = ... [2]

(iii) B is the mid-point of the line AC.

Find the co-ordinates of C.

(................... , ...) [2]

(iv) Find the equation of the straight line that passes through A and B.

.... [3]

(v) The straight line that passes through A and B cuts the y-axis at D.

Write down the co-ordinates of D.

(................... , ...) [1]

[Nov/2018/P43/Q1(b)]

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4. (a) The equation of a straight line is $2y = 3x + 4$.

(i) Find the gradient of this line.

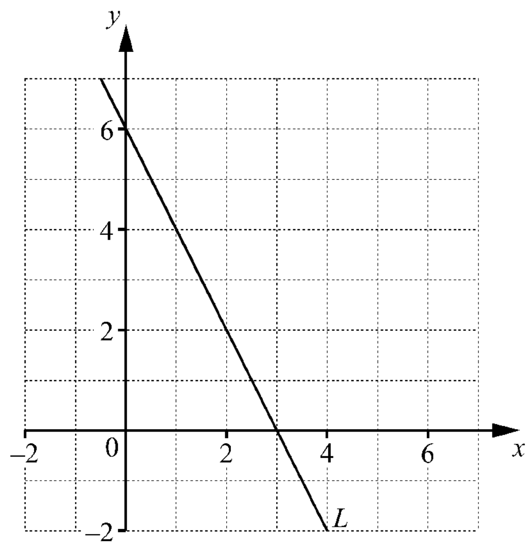
.... [1]

(ii) Find the co-ordinates of the point where the line crosses the y-axis.

(.................... , ...) [1]

(b) The diagram shows a straight line L.

(i) Find the equation of line L. [3]



.... [3]

(ii) Find the equation of the line perpendicular to line L that passes through (9, 3).

.... [3]

(c) A is the point (8, 5) and B is the point (–4, 1).

(i) Calculate the length of AB.

.... [3]

(ii) Find the co-ordinates of the midpoint of *AB*.

(.................... , ...) [2]

[June/2019/P42/Q4]

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5. A straight line joins the points *A* (–2, –3) and *C* (1, 9).

(a) Find the equation of the line *AC* in the form $y = mx + c$.

y = ... [3]

(b) Calculate the acute angle between *AC* and the *x*-axis.

.... [2]

(c) *ABCD* is a kite, where *AC* is the longer diagonal of the kite.
B is the point (3.5, 2).

(i) Find the equation of the line *BD* in the form $y = mx + c$.

y = ... [3]

(ii) The diagonals AC and BD intersect at (–0.5, 3).

Work out the co-ordinates of D.

(...................... , ...) [2]

[June/2019/P43/Q7]

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6. A rhombus $ABCD$ has a diagonal AC where A is the point (–3, 10) and C is the point (4, –4).

(i) Calculate the length AC.

.... [3]

(ii) Show that the equation of the line AC is $y = -2x + 4$.

[2]

(iii) Find the equation of the line BD.

.... [4]

[June/2020/P41/Q10(a)]

ANSWERS

Topic 18 - Coordinate Geometry

1. $y = \frac{1}{2}x + 1$

Gradient of line $= \frac{1}{2}$

(Grad. of line) $\times$ (Grad. of perp. line) $= -1$

$\Rightarrow \frac{1}{2} \times$ (Grad. of perp. line) $= -1$

$\Rightarrow$ (Grad. of perp. line) $= -2$

Equation of perpendicular line : $y = -2x + c$

Using point, (1, 3),

$3 = -2(1) + c \Rightarrow c = 5$

$\therefore$ Equation of perpendicular line is,

$y = -2x + 5$

2. **(a)** Co-ordinates of $A = (5, 6)$.

(b) Using points (0, 3) and (5, – 1),

gradient, $m = \frac{-1-3}{5-0} = -\frac{4}{5}$

From graph, y-intercept, $c = 3$

$\therefore$ Equation of line l : $y = -\frac{4}{5}x + 3$

(c) Parallel lines have the same gradients.

$\therefore$ Gradient of parallel line $= -\frac{4}{5}$

Equation of line is, $y = -\frac{4}{5}x + c$

As this line passess through B,
therefore its y-intercept is, $c = -2$

$\therefore$ Equation of parallel line is, $y = -\frac{4}{5}x - 2$

(d) **(i)** (Grad. of perp. line) $\times$ (Grad. of l) $= -1$

$\Rightarrow$ (Grad. of perp. line) $\times -\frac{4}{5} = -1$

$\Rightarrow$ Grad. of perp. line $= \frac{5}{4}$

Equation of perp. line is, $y = \frac{5}{4}x + c$

Using point, $C(8, 14)$,

$14 = \frac{5}{4}(8) + c$

$\Rightarrow 14 = 10 + c \Rightarrow c = 4$

$\therefore$ Equation of perp. line is, $y = \frac{5}{4}x + 4$

(ii) $A(5, 6)$, $C(8, 14)$

$\therefore AC = \sqrt{(8-5)^2 + (14-6)^2}$

$= \sqrt{(3)^2 + (8)^2}$

$= \sqrt{73} = 8.54$ units

(iii) $B(0, -2)$, $C(8, 14)$

Mid-point of $BC = \left(\frac{0+8}{2}, \frac{-2+14}{2}\right)$

$= (4, 6)$

3. **(i)** $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= \begin{pmatrix} 5 \\ 6 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

(ii) $\left|\overrightarrow{AB}\right| = \sqrt{(2)^2 + (4)^2}$

$= \sqrt{20} = 4.47$ units

(iii) Let the coordinates of C be (x, y),

mid-point of AC = point B

$\Rightarrow \left(\frac{x+3}{2}, \frac{y+2}{2}\right) = (5, 6)$

$\Rightarrow \frac{x+3}{2} = 5$, $\frac{y+2}{2} = 6$

$\Rightarrow x + 3 = 10$, $y + 2 = 12$

$x = 7$, $y = 10$

$\therefore$ Coordinates of C are (7, 10)

(iv) Gradient of $AB = \frac{6-2}{5-3}$

$= \frac{4}{2} = 2$

Equation of AB is : $y = 2x + c$

Using $A(3, 2)$;

$2 = 2(3) + c \Rightarrow 2 = 6 + c \Rightarrow c = -4$

$\therefore$ Equation of line AB is : $y = 2x - 4$

(v) Equation of line AB : $y = 2x - 4$

y-intercept of the line is, $c = -4$

$\therefore$ co-ordinates of D are $(0, -4)$

4. (a) (i) $2y = 3x + 4 \;\Rightarrow\; y = \frac{3}{2}x + 2$

$\therefore$ Gradient $= \frac{3}{2}$

(ii) For y-intercept, $x = 0$,

$\Rightarrow\; y = \frac{3}{2}(0) + 2 = 2$

$\therefore$ co-ordinates are $(0,\ 2)$

(b) (i) Using points (0, 6) and (3, 0) from graph,

Gradient $= \frac{0-6}{3-0} = \frac{-6}{3} = -2$

y-intercept, $c = 6$

$\therefore$ Equation of L is, $y = -2x + 6$

(ii) Gradient of $L = -2$

$\Rightarrow$ Gradient of perpendicular to $L = \frac{1}{2}$

Equation of line perp. to L, $y = \frac{1}{2}x + c$

Using (9, 3), $3 = \frac{1}{2}(9) + c \;\Rightarrow\; c = -\frac{3}{2}$

$\therefore$ Eq. of line perp. to L is, $y = \frac{1}{2}x - \frac{3}{2}$

(c) (i) $AB = \sqrt{(8-(-4))^2 + (5-1)^2}$

$= \sqrt{(12)^2 + (4)^2}$

$= \sqrt{160} \approx 12.6$ units

(ii) Mid-point of $AB = \left(\frac{8+(-4)}{2},\ \frac{5+1}{2}\right)$

$= \left(\frac{4}{2},\ \frac{6}{2}\right) = (2,\ 3)$

5. (a) Gradient of $AC = \frac{9-(-3)}{1-(-2)} = \frac{12}{3} = 4$

Equation of AC is, $y = 4x + c$

Using (1, 9), $9 = 4(1) + c \;\Rightarrow\; c = 5$

$\therefore$ Equation of AC is, $y = 4x + 5$

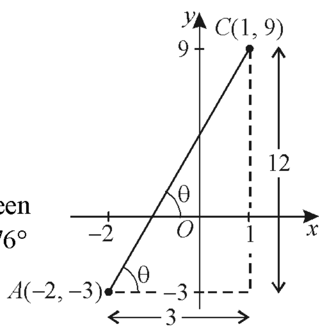
(b) From figure,

$\tan\theta = \frac{12}{3}$

$\Rightarrow\; \theta = \tan^{-1}(4)$

$\approx 76°$

Acute angle between AC and x-axis = 76°



(c) (i) As $ABCD$ is a kite, diagonal BD is perpendicular to diagonal AC

$\Rightarrow$ Gradient of $BD = -\frac{1}{4}$

Equation of BD : $y = -\frac{1}{4}x + c$

Using $B(3.5,\ 2)$,

$2 = -\frac{1}{4}(3.5) + c \;\Rightarrow\; c = 2 + \frac{3.5}{4} = \frac{23}{8}$

$\therefore$ Equation of BD is, $y = -\frac{1}{4}x + \frac{23}{8}$

(ii) Let coordinates of D be $(x,\ y)$,

$(-0.5,\ 3)$ is the mid-point of BD

$\Rightarrow\; \left(\frac{3.5+x}{2},\ \frac{2+y}{2}\right) = (-0.5,\ 3)$

$\Rightarrow\; \frac{3.5+x}{2} = -0.5 \quad , \quad \frac{2+y}{2} = 3$

$3.5 + x = -1 \quad , \quad 2 + y = 6$

$x = -4.5 \quad , \quad y = 4$

$\therefore$ D is $(-4.5,\ 4)$

6. (i) $AC = \sqrt{(4-(-3))^2 + (-4-10)^2}$

$= \sqrt{(7)^2 + (-14)^2}$

$= \sqrt{245} \approx 15.7$ units

(ii) Gradient of $AC = \frac{-4-10}{4-(-3)} = -\frac{14}{7} = -2$

Equation of AC is, $y = -2x + c$

Substitute $A(-3,\ 10)$,

$10 = -2(-3) + c \;\Rightarrow\; 10 = 6 + c \;\Rightarrow\; c = 4$

$\therefore$ Equation of AC is, $y = -2x + 4$

(iii) Line BD is the 2nd diagonal of the rhombus. BD is also the perpendicular bisector of AC

Midpoint of $AC = \left(\frac{-3+4}{2},\ \frac{10-4}{2}\right)$

$= \left(\frac{1}{2},\ 3\right)$

Gradient of $AC = -2$

Now,

(Grad. of AC)$\times$(Grad. of BD) $= -1$

$\Rightarrow\; -2 \times$ (Grad. of BD) $= -1$

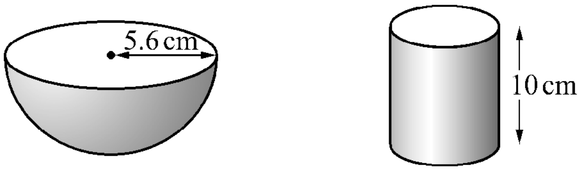
$\Rightarrow$ Grad. of $BD = \frac{1}{2}$

Equation of BD is, $y = \frac{1}{2}x + c$

TOPIC 24

Mensuration

1. (a)



The diagram shows a hemispherical bowl of radius 5.6 cm and a cylindrical tin of height 10 cm.

(i) Show that the volume of the bowl is 368 cm^3, correct to the nearest cm^3.

[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

[2]

(ii) The tin is completely full of soup.

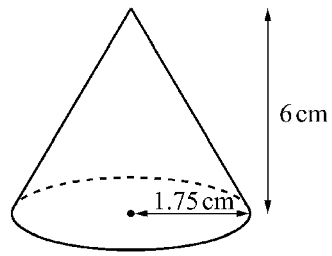
When all the soup is poured into the empty bowl, 80% of the volume of the bowl is filled.

Calculate the radius of the tin.

.... cm [4]

(b) The diagram shows a cone with radius 1.75 cm and height 6 cm.

(i) Calculate the total surface area of the cone.
[The curved surface area, A, of a cone with radius r and slant height l is $A = \pi r l$.]



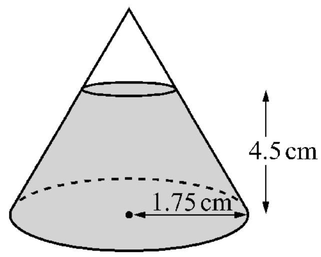
.... cm^2 [5]

(ii) The cone contains salt to a depth of 4.5 cm.

The top layer of the salt forms a circle that is parallel to the base of the cone.

(a) Show that the volume of the salt inside the cone is $18.9\ cm^3$, correct to 1 decimal place.

[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]



[4]

(b) The salt is removed from the cone at a constant rate of $200\ mm^3$ per second.

Calculate the time taken for the cone to be completely emptied.

Give your answer in seconds, correct to the nearest second.

.... s [3]

[June/2019/P43/Q4]

2. (a) (i) Calculate the **external curved** surface area of a cylinder with radius 8 m and height 19 m.

.... m^2 [2]

(ii) This surface is painted at a cost of $0.85 per square metre.

Calculate the cost of painting this surface.

$ .... [2]

(b) A solid metal sphere with radius 6 cm is melted down and all of the metal is used to make a solid cone with radius 8 cm and height h cm.

(i) Show that $h = 13.5$.

[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

[2]

(ii) Calculate the slant height of the cone.

.... cm [2]

(iii) Calculate the curved surface area of the cone.

[The curved surface area, A, of a cone with radius r and slant height l is $A = \pi r l$.]

.... cm^2 [1]

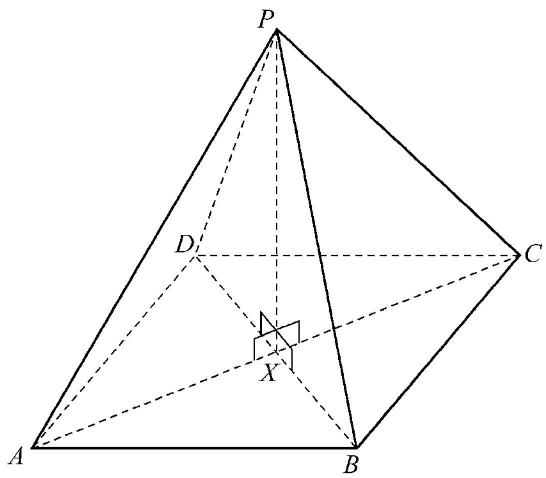
(c) Two cones are mathematically similar.
The total surface area of the smaller cone is 80 cm^2.
The total surface area of the larger cone is 180 cm^2.
The volume of the smaller cone is 168 cm^3.

Calculate the volume of the larger cone.

.... cm^3 [3]

(d) The diagram shows a pyramid with a square base *ABCD*.
DB = 8 cm.
P is vertically above the centre, *X*, of the base and *PX* = 5 cm

Calculate the angle between *PB* and the base *ABCD*..

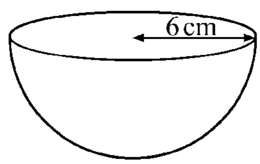


.... [3]

[Nov/2019/P41/Q4]

3. The diagram shows a hemisphere with radius 6 cm.
Calculate the volume. Give the units of your answer.

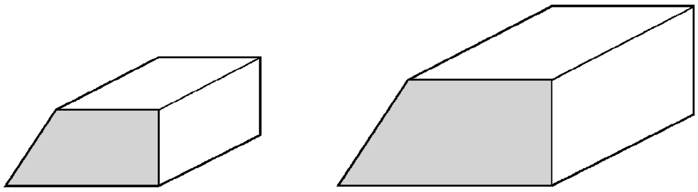
[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]



.... [3]

[Nov/2019/P42/Q4(a)]

4.



The diagram shows two mathematically similar solid metal prisms.
The volume of the smaller prism is 648 cm^3 and the volume of the larger prism is 2187 cm^3.
The area of the cross-section of the smaller prism is 36 cm^2.

(i) Calculate the area of the cross-section of the larger prism.

.... cm^2 [3]

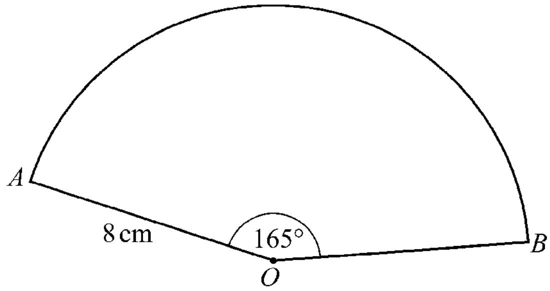
(ii) The larger prism is melted down into a sphere.
Calculate the radius of the sphere.
[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

.... cm [3]

[Nov/2019/P43/Q6(b)]

5. The diagram shows a sector of a circle with centre O, radius 8 cm and sector angle 165°.

(a) Calculate the total perimeter of the sector.



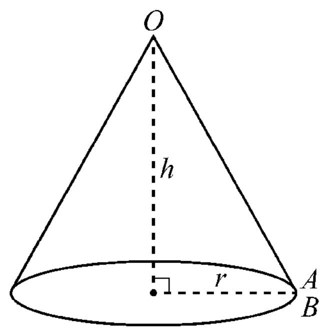
.... cm [3]

(b) The surface area of a sphere is the same as the area of the sector.
Calculate the radius of the sphere.
[The surface area, A, of a sphere with radius r is $A = 4\pi r^2$.]

.... cm [4]

(c) A cone is made from the sector by joining OA to OB.

(i) Calculate the radius, r, of the cone.



r = ... cm [2]

(ii) Calculate the volume of the cone.
[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

.... cm^3 [4]

[June/2020/P41/Q9]

6. (a) A cylinder with radius 6 cm and height h cm has the same volume as a sphere with radius 4.5 cm.

Find the value of h.

[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

h = ... [3]

(b) A solid metal cube of side 20 cm is melted down and made into 40 solid spheres, each of radius r cm.

Find the value of r.

r = ... [3]

(c) A solid cylinder has radius x cm and height $\frac{7x}{2}$ cm.

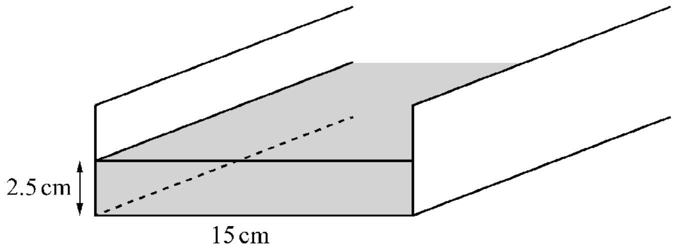
The surface area of a sphere with radius R cm is equal to the total surface area of the cylinder. Find an expression for R in terms of x.

[The surface area, A, of a sphere with radius r is $A = 4\pi r^2$.]

R = ... [3]

[June/2020/P42/Q8(b,c,d)]

7.



Water flows at a speed of 20 cm/s along a rectangular channel into a lake.
The width of the channel is 15 cm.
The depth of the water is 2.5 cm.
Calculate the amount of water that flows from the channel into the lake in 1 hour.
Give your answer in litres.

.... litres [4]

[June/2020/P43/Q6(b)]

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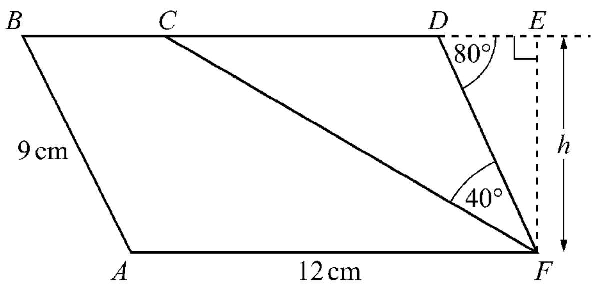
8. (a)



ABDF is a parallelogram and *BCDE* is a straight line.
AF = 12 cm, *AB* = 9 cm, angle *CFD* = 40° and angle *FDE* = 80°.

(i) Calculate the height, *h*, of the parallelogram.

h = ... cm [2]

ANSWERS

Topic 24 - Mensuration

1. (a) (i) Volume of bowl $= \frac{1}{2}\left(\frac{4}{3}\pi(5.6)^3\right)$

$= 367.809 \approx 368 \text{ cm}^3$

(ii) Volume of tin = 80% of volume of bowl

$\Rightarrow \pi r^2(10) = \frac{80}{100} \times 368$

$\Rightarrow 10\pi r^2 = 294.4$

$\Rightarrow r^2 = \frac{294.4}{10\pi}$

$\Rightarrow r^2 = 9.371 \Rightarrow r = 3.06$ cm.

(b) (i) Using Pythagoras theorem, slant height of the cone is, $l = \sqrt{1.75^2 + 6^2}$

$= \sqrt{39.0625} = 6.25$ cm

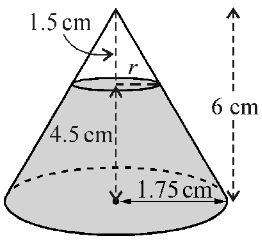
Total surface area of the cone

= area of base + curved surface area

$= \pi(1.75)^2 + \pi(1.75)(6.25)$

$= 9.621 + 34.36 = 43.981 \approx 44.0 \text{ cm}^2$

(ii) (a)



Let radius of top smaller cone be r cm

Height of smaller cone $= 6 - 4.5$

$= 1.5$ cm

Using rule of similar triangles,

$\frac{r}{1.75} = \frac{1.5}{6}$

$\Rightarrow r = \frac{1.5}{6} \times 1.75 = 0.4375$ cm

Volume of salt = vol. of larger cone − vol. of smaller cone

$= \frac{1}{3}\pi(1.75)^2(6) - \frac{1}{3}\pi(0.4375)^2(1.5)$

$= 19.242 - 0.3007 \approx 18.9 \text{ cm}^3$ (to 1 dp)

(b) Volume of sand in $\text{mm}^3 = 18.9 \times 10^3$

$= 18900 \text{ mm}^3$

Rate of removing sand $= 200 \text{ mm}^3/\text{s}$

∴ Time taken to empty the cone

$= \frac{18900}{200} = 94.5 \text{ s} \approx 95$ s.

2. (a) (i) Curved surface area $= 2\pi rh$

$= 2\pi(8)(19) \approx 955 \text{ m}^2$

(ii) Cost of painting $= \$0.85 \times 955$

$= \$811.75$

(b) (i) Volume of sphere melted = volume of cone

$\Rightarrow \frac{4}{3}\pi(6)^3 = \frac{1}{3}\pi(8)^2 h$

$\Rightarrow 288\pi = \frac{64}{3}\pi h$

$\Rightarrow h = 288\pi \times \frac{3}{64\pi} = 13.5$ cm.

(ii) Let slant height be l.

Using Pythagoras theorem,

$l = \sqrt{8^2 + 13.5^2}$

$= \sqrt{246.25} \approx 15.7$ cm

(iii) Curved surface area

$= \pi(8)(15.7)$

$= 394.58 \approx 395 \text{ cm}^2$

(c) $\frac{\text{Area}_{\text{small}}}{\text{Area}_{\text{large}}} = \left(\frac{\text{Length}_{\text{small}}}{\text{Length}_{\text{large}}}\right)^2$

$\Rightarrow \frac{80}{180} = \left(\frac{\text{Length}_{\text{small}}}{\text{Length}_{\text{large}}}\right)^2$

$\Rightarrow \frac{\text{Length}_{\text{small}}}{\text{Length}_{\text{large}}} = \sqrt{\frac{80}{180}}$

$\Rightarrow \frac{\text{Length}_{\text{small}}}{\text{Length}_{\text{large}}} = \frac{2}{3}$

Now, $\frac{\text{Volume}_{\text{small}}}{\text{Volume}_{\text{large}}} = \left(\frac{\text{Length}_{\text{small}}}{\text{Length}_{\text{large}}}\right)^3$

$\Rightarrow \frac{168}{\text{Volume}_{\text{large}}} = \left(\frac{2}{3}\right)^3$

$\Rightarrow \frac{168}{\text{Volume}_{\text{large}}} = \frac{8}{27}$

$\Rightarrow 168 \times 27 = 8(\text{Volume}_{\text{large}})$

$\Rightarrow \text{Volume}_{\text{large}} = \frac{168 \times 27}{8} = 567 \text{ cm}^3$

(d) In ΔPBX, the angle between PB and the base $ABCD$ is $P\hat{B}X$

$XB = \frac{1}{2}DB = \frac{1}{2}(8) = 4$ cm

Now, $\tan P\hat{B}X = \frac{PX}{XB}$

$\Rightarrow \tan P\hat{B}X = \frac{5}{4} \Rightarrow P\hat{B}X = 51.3°$

3. Volume $= \frac{1}{2}\left(\frac{4}{3}\pi(6)^3\right)$

$= 452.39 \approx 452$ cm^3

4. **(i)** $\frac{\text{Volume}_{\text{large}}}{\text{Volume}_{\text{small}}} = \left(\frac{\text{Length}_{\text{large}}}{\text{Length}_{\text{small}}}\right)^3$

$\Rightarrow \frac{2187}{648} = \left(\frac{\text{Length}_{\text{large}}}{\text{Length}_{\text{small}}}\right)^3$

$\Rightarrow \frac{\text{Length}_{\text{large}}}{\text{Length}_{\text{small}}} = \sqrt[3]{\frac{2187}{648}}$

$\Rightarrow \frac{\text{Length}_{\text{large}}}{\text{Length}_{\text{small}}} = \frac{3}{2}$

Now, $\frac{\text{Area}_{\text{large}}}{\text{Area}_{\text{small}}} = \left(\frac{\text{Length}_{\text{large}}}{\text{Length}_{\text{small}}}\right)^2$

$\Rightarrow \frac{\text{Area}_{\text{large}}}{36} = \left(\frac{3}{2}\right)^2$

$\Rightarrow \text{Area}_{\text{large}} = \frac{9}{4} \times 36 = 81$ cm^2

(ii) Volume of sphere = volume of large prism

$\frac{4}{3}\pi r^3 = 2187$

$r^3 = 2187 \times \frac{3}{4\pi}$

$r^3 = 522.108$

$r \approx 8.05$ cm

5. **(a)** Perimeter of sector = Arc length $+ OA + OB$

$= \frac{165°}{360°} \times 2\pi(8) + 8 + 8$

$= 23.04 + 16$

$= 39.04 \approx 39.0$ cm

(b) Surface area of sphere = area of the sector

$\Rightarrow 4\pi r^2 = \frac{165°}{360°} \times (\pi)(8)^2$

$\Rightarrow r^2 = \frac{165°}{360°} \times (\pi)(8)^2 \times \frac{1}{4\pi}$

$\Rightarrow r^2 = 7.333 \Rightarrow r = 2.71$ cm.

(c) **(i)** Circumference of base of cone = arc length of sector

$\Rightarrow 2\pi r = \frac{165°}{360°} \times 2(\pi)(8)$

$\Rightarrow r = \frac{165°}{360°} \times (8) \Rightarrow r = 3.67$

$\therefore$ Radius of the cone = 3.67 cm

(ii) By pythagoras theorem, height of cone is, $h = \sqrt{(8)^2 - (3.67)^2}$

$= \sqrt{50.5311} = 7.11$ cm

Volume of cone $= \frac{1}{3}\pi(3.67)^2(7.11)$

$= 100.28 \approx 100$ cm^3

6. **(a)** Volume of cylinder = volume of sphere

$\Rightarrow \pi(6)^2 h = \frac{4}{3}(\pi)(4.5)^3$

$\Rightarrow 36\pi h = 121.5\pi$

$\Rightarrow h = \frac{121.5\pi}{36\pi} = 3.375$ cm

(b) Volume of 40 spheres = Volume of cube

$\Rightarrow 40\left(\frac{4}{3}\pi r^3\right) = 20^3$

$\Rightarrow \frac{160}{3}\pi r^3 = 8000$

$\Rightarrow r^3 = 8000 \times \frac{3}{160\pi}$

$\Rightarrow r^3 = 47.746 \Rightarrow r = 3.63$ cm

(c) Surface area of sphere = total surface area of cylinder

$\Rightarrow 4\pi R^2 = 2\pi x^2 + 2\pi(x)\left(\frac{7x}{2}\right)$

$\Rightarrow 4\pi R^2 = 2\pi x^2 + 7\pi x^2$

$\Rightarrow 4\pi R^2 = 9\pi x^2$

$\Rightarrow R^2 = \frac{9\pi x^2}{4\pi} \Rightarrow R = \sqrt{\frac{9x^2}{4}} = \frac{3x}{2}$ cm.

7. Speed of water flow = 20 cm/s

Area of cross section $= 15 \times 2.5 = 37.5$ cm^2

Volume of water flowing into the lake in 1 second $= 37.5 \times 20 = 750$ cm^3

$\therefore$ Amount of water that flows in 1 hour

$= 750 \times 60 \times 60$

$= 2700\,000$ cm^3

$= \frac{2700\,000}{1000} = 2700$ litres

8. **(a)** **(i)** In ΔDEF, $FD = 9$ cm

$\sin 80° = \frac{h}{9} \Rightarrow h = 9\sin 80° = 8.86$ cm

(ii) $D\hat{C}F + 40° = 80°$ (ext. ∠ of Δ = sum of opp. interior angles)

$\Rightarrow D\hat{C}F = 80° - 40° = 40°$

As, $C\hat{F}D = D\hat{C}F = 40°$ (two equal ∠s)

$\therefore$ ΔCDF is an isosceles triangle

(iii) $CD = DF = 9$ cm (ΔCDF is isosceles)

$\Rightarrow BC = 12 - 9 = 3$ cm

Area of trapezium $ABCF$

$= \frac{1}{2}(12+3)(8.86) = 66.45 \text{ cm}^2$

(b) $A\hat{D}C = 90°$ (right angle in semicircle)

$A\hat{C}D = 21°$ (angles in the same segment)

Now, in ΔACD, $\cos 21° = \frac{12}{AC}$

$\Rightarrow AC = \frac{12}{\cos 21°} = 12.85$ cm

Radius of circle, $r = \frac{12.85}{2} = 6.425$ cm

$\therefore$ Area of circle $= \pi(6.425)^2$

$= 129.687 \approx 130 \text{ cm}^2$

(c) Perimeter of square = perimeter of sector

$\Rightarrow 4 \times 8 = 9.5 + 9.5 +$ arc length of sector

$\Rightarrow 32 = 19 + \frac{x°}{360°} \times 2\pi(9.5)$

$\Rightarrow 32 = 19 + \frac{19\pi x°}{360°}$

$\Rightarrow 13 = \frac{19\pi x°}{360°}$

$\Rightarrow x° = 13 \times \frac{360°}{19\pi} \Rightarrow x° = 78.4°$

9. **(a)** Volume of cuboid $= 8 \times 5 \times 11$

$= 440 \text{ cm}^3$

(b) We can decide this by finding the length of the diagonal AG.

Using Pythagoras theorem on ΔABC,

$AC^2 = 8^2 + 5^2 = 89$ cm

Again by Pythagoras theorem on ΔAGC,

$AG = \sqrt{AC^2 + 11^2}$

$= \sqrt{89 + 11^2} = \sqrt{210} = 14.5$ cm

$\therefore$ Yes, pencil fits completely inside the cuboid.

(c) **(i)** In ΔABC, $\tan C\hat{A}B = \frac{5}{8}$

$\Rightarrow C\hat{A}B = \tan^{-1}\left(\frac{5}{8}\right) = 32.0°$

(ii) From (b), $AC = \sqrt{89} = 9.434$ cm

In ΔAGC, $\tan G\hat{A}C = \frac{11}{AC}$

$\Rightarrow G\hat{A}C = \tan^{-1}\left(\frac{11}{9.434}\right) = 49.4°$

10. **(a)** Total surface area of cone = total surface area of hemisphere

$\Rightarrow \pi(2.4)^2 + \pi(2.4)(6.3) = \pi R^2 + \frac{1}{2}(4\pi R^2)$

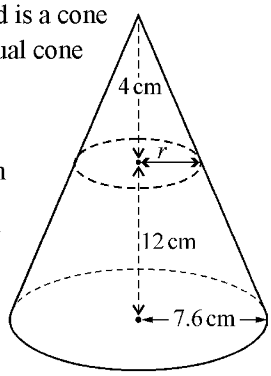
$\Rightarrow 5.76\pi + 15.12\pi = 3\pi R^2$

$\Rightarrow 20.88\pi = 3\pi R^2$

$\Rightarrow R^2 = \frac{20.88\pi}{3\pi}$

$\Rightarrow R^2 = 6.96 \Rightarrow R = 2.64$ cm

(b) The top section removed is a cone that is similar to the actual cone



$\frac{r}{7.6} = \frac{4}{16}$

$\Rightarrow r = \frac{4}{16} \times 7.6 = 1.9$ cm

$\therefore$ Radius of top section cone = 1.9 cm

Vol. of remaining solid

= Vol. of actual cone − Vol. of top section cone

$= \frac{1}{3}\pi(7.6)^2(16) - \frac{1}{3}(\pi)(1.9)^2(4)$

$= 967.78 - 15.12 = 952.66 \approx 953 \text{ cm}^3$

11. **(a)** In ΔABC, by Pythagoras theorem,

$BC = \sqrt{20^2 - 13^2} = \sqrt{231} \approx 15.2$ cm

Total surface area

$= 2\left(\frac{1}{2}(13)(15.2)\right) + (20 \times 24) + (24 \times 15.2) + (13 \times 24)$

$= 197.6 + 480 + 364.8 + 312$

$= 1354.4 \text{ cm}^2 \approx 1350 \text{ cm}^2$

(b) Volume = area of triangle × prism length

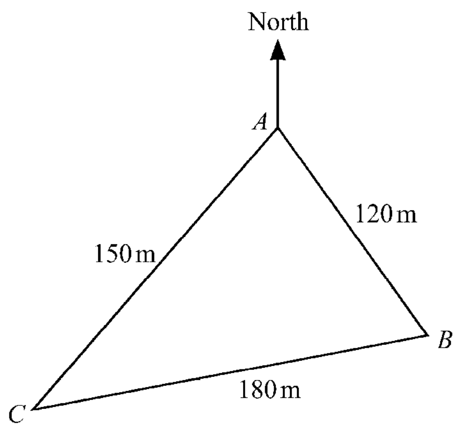
$= \frac{1}{2}(13)(15.2) \times 24$

$= 2371.2 \approx 2370 \text{ cm}^3$

TOPIC 25

Trigonometry and Bearings

1. The diagram shows a triangular field, *ABC*, on horizontal ground.



(a) Olav runs from *A* to *B* at a constant speed of 4 m/s and then from *B* to *C* at a constant speed of 3 m/s.

He then runs at a constant speed from *C* to *A*.

His average speed for the whole journey is 3.6 m/s.

Calculate his speed when he runs from *C* to *A*.

.... m/s [3]

(b) Use the cosine rule to find angle *BAC*.

Angle *BAC* = ... [4]

(c) The bearing of *C* from *A* is 210°.

(i) Find the bearing of *B* from *A*.

.... [1]

(ii) Find the bearing of A from B.

.... [2]

(d) D is the point on AC that is nearest to B. Calculate the distance from D to A.

.... m [2]

[Nov/2019/P41/Q5]

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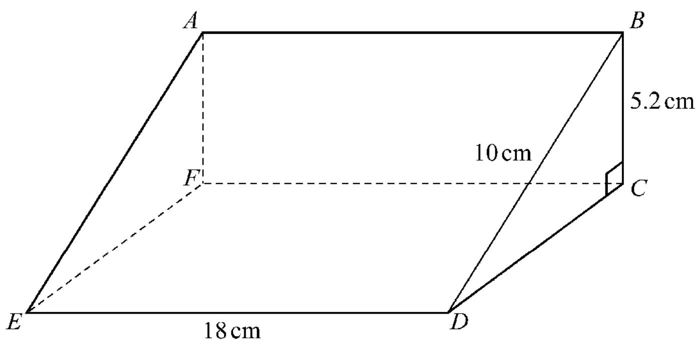
2. The diagram shows a prism $ABCDEF$. The cross-section is a right-angled triangle BCD.

$BD = 10$ cm, $BC = 5.2$ cm and $ED = 18$ cm.



(i) (a) Work out the volume of the prism.

.... cm^3 [6]

(b) Calculate angle BEC.

Angle BEC = ... [4]

(ii) The point G lies on the line ED and $GD = 7$ cm.
Work out angle BGE.

Angle BGE = ... [3]

[Nov/2019/P42/Q4(b)]

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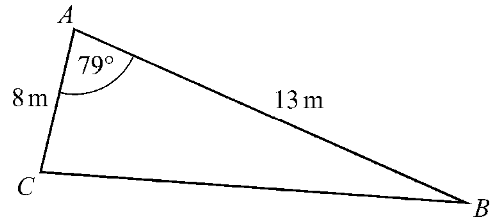
3. (a) The diagram shows triangle ABC.

(i) Use the cosine rule to calculate BC.



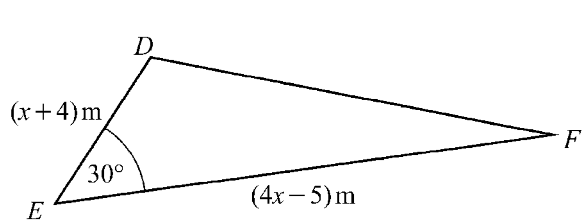
BC = ... m [4]

(ii) Use the sine rule to calculate angle ACB.

Angle ACB = ... [3]

(b) The area of triangle DEF is 70 m^2.

(i) Show that $4x^2 + 11x - 300 = 0$.



[4]

(ii) Use the quadratic formula to solve $4x^2 + 11x - 300 = 0$.
Show all your working and give your answers correct to 2 decimal places.

x = ... or x = ... [4]

(iii) Find the length of DE.

DE = ... m [1]

[Nov/2019/P42/Q6]

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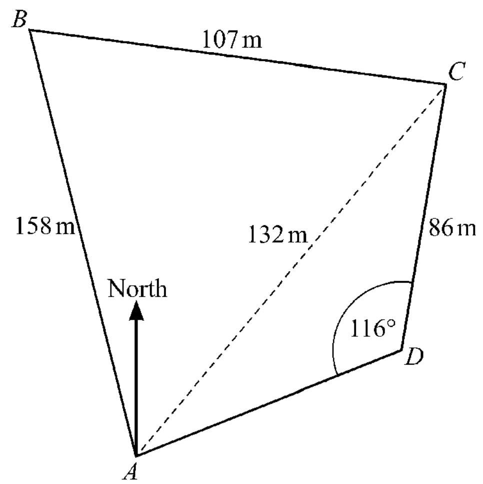
4. The diagram shows a field, $ABCD$, on horizontal ground.

(a) There is a vertical post at C.
From B, the angle of elevation of the top of the post is 19°.
Find the height of the post.



.... m [2]

(b) Use the cosine rule to find angle BAC.

Angle BAC = ... [4]

(c) Use the sine rule to find angle *CAD*.

Angle *CAD* = ... [3]

(d) Calculate the area of the field.

.... m^2 [3]

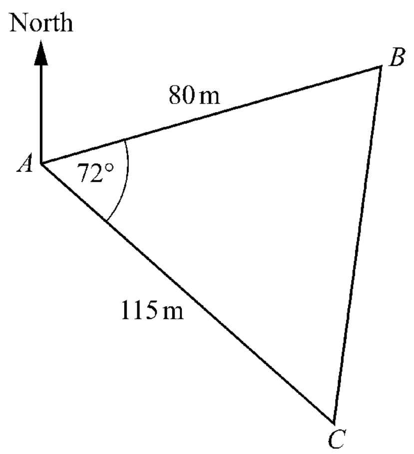
(e) The bearing of *D* from *A* is 070°.

Find the bearing of *A* from *C*.

.... [2]

[Nov/2019/P43/Q4]

5. The diagram shows the positions of three points *A*, *B* and *C* in a field.



(a) Show that *BC* is 118.1 m, correct to 1 decimal place.

[3]

(b) Calculate angle ABC.

Angle ABC = ... [3]

(c) The bearing of C from A is 147°.

Find the bearing of

(i) A from B,

.... [3]

(ii) B from C.

.... [2]

(d) Mitchell takes 35 seconds to run from A to C.

Calculate his average running speed in kilometres per hour.

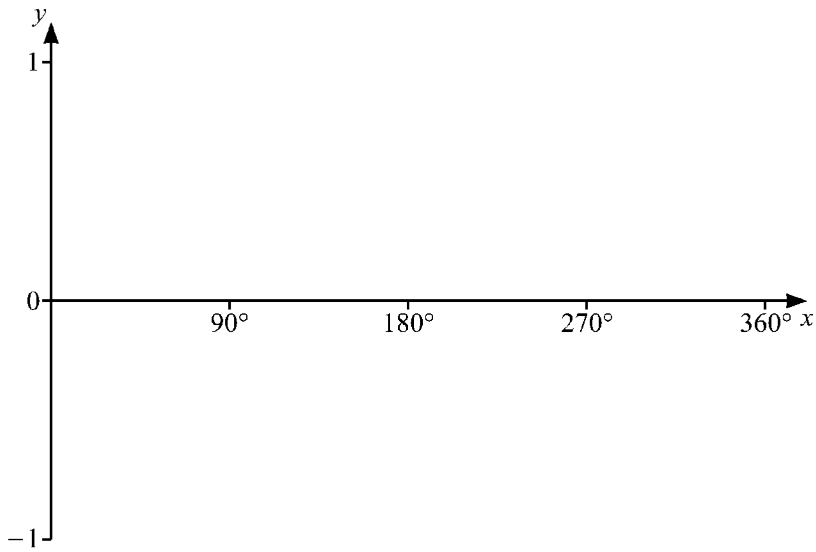
.... km/h [3]

(e) Calculate the shortest distance from point B to AC.

.... m [3]

[June/2020/P41/Q7]

6. (a) (i) On the axes, sketch the graph of $y = \sin x$ for $0° \le x \le 360°$. [2]



(ii) Describe fully the symmetry of the graph of $y = \sin x$ for $0° \le x \le 360°$.

....

.... [2]

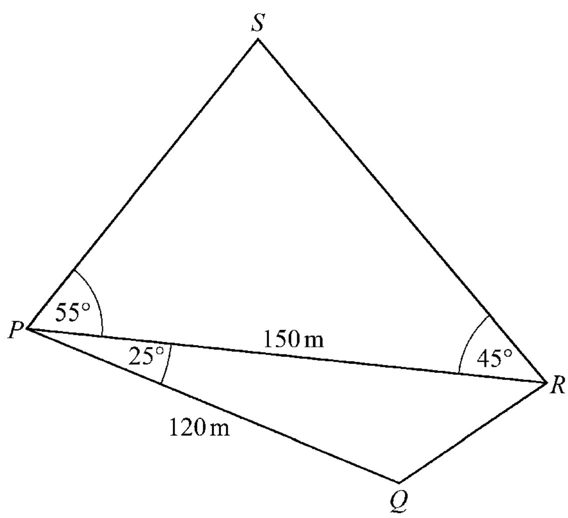
(b) Solve $4\sin x - 1 = 2$ for $0° \le x \le 360°$.

$x =$ .... and $x =$ .... [3]

[June/2020/P41/Q8(a,b)]

7. The diagram shows two triangles.

(a) Calculate *QR*. [3]



QR = ... m

(b) Calculate RS.

RS = ... m [4]

(c) Calculate the total area of the two triangles.

.... m^2 [3]

[June/2020/P42/Q4]

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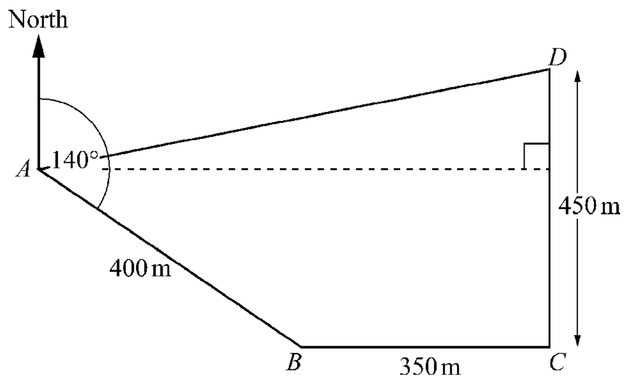
8. The diagram shows a field $ABCD$.
The bearing of B from A is 140°.
C is due east of B and D is due north of C.
AB = 400 m, BC = 350 m and CD = 450 m.



(a) Find the bearing of D from B.

.... [2]

(b) Calculate the distance from D to A.

.... m [6]

(c) Jono runs around the field from A to B, B to C, C to D and D to A.

He runs at a speed of 3 m/s.

Calculate the total time Jono takes to run around the field.

Give your answer in minutes and seconds, correct to the nearest second.

.... min .... s [4]

[June/2020/P42/Q5]

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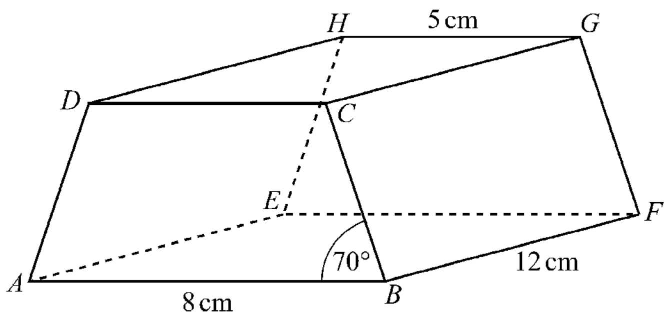
9. The diagram shows a prism with a rectangular base, *ABFE*.

The cross-section, *ABCD*, is a trapezium with $AD = BC$.

$AB = 8$ cm, $GH = 5$ cm, $BF = 12$ cm and angle $ABC = 70°$.



(a) Calculate the total surface area of the prism.

.... cm^2 [6]

(b) The perpendicular from G onto EF meets EF at X.

(i) Show that $EX = 6.5$ cm.

[1]

(ii) Calculate AX.

AX = ... cm [2]

(iii) Calculate the angle between the diagonal AG and the base $ABFE$.

.... [2]

[Nov/2020/P42/Q9]

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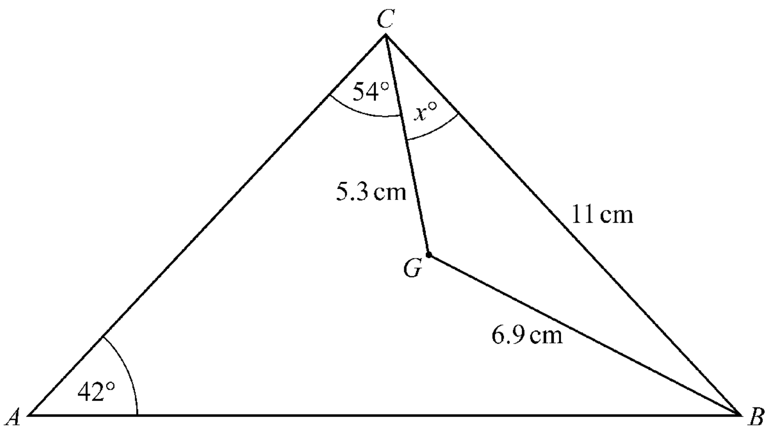
10. The diagram shows triangle ABC with point G inside.

$CB = 11$ cm, $CG = 5.3$ cm and $BG = 6.9$ cm.

Angle $CAB = 42°$ and angle $ACG = 54°$.



(i) Calculate the value of x.

x = ... [4]

ANSWERS

Topic 25 - Trigonometry and Bearings

1. **(a)** Total distance for the whole journey

$= 150 + 120 + 180 = 450$ m

Total time taken for the whole journey

$= \dfrac{\text{total distance}}{\text{average speed}} = \dfrac{450}{3.6}$

$= 125$ seconds

Time taken from A to $B = \dfrac{120}{4} = 30$ seconds

Time taken from B to $C = \dfrac{180}{3} = 60$ seconds

$\Rightarrow$ Time taken from C to $A = 125 - 30 - 60$

$= 35$ seconds

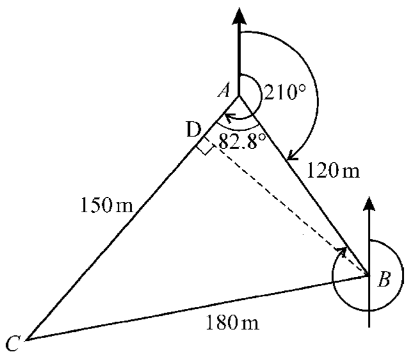
$\therefore$ Speed from C to $A = \dfrac{150}{35} = 4.286$

≈ 4.29 m/s

(b) $\cos B\hat{A}C = \dfrac{150^2 + 120^2 - 180^2}{2 \times 150 \times 120}$

$\Rightarrow \cos B\hat{A}C = \dfrac{4500}{36000} \Rightarrow B\hat{A}C = 82.8°$

(c)



(i) Bearing of B from $A = 210° - 82.8°$

$= 127.2°$

(ii) Bearing of A from $B = 180° + 127.2°$

$= 307.2°$

(d) D is nearest to B when BD is perpendicular to AC as shown in the figure above.

In ΔABD, $\cos 82.8° = \dfrac{AD}{120}$

$\Rightarrow AD = 120 \cos 82.8° \Rightarrow AD = 15.0$ m

2. **(i)** **(a)** In ΔBDC, using Pythagoras theorem,

$DC = \sqrt{10^2 - 5.2^2} = \sqrt{72.96} \approx 8.54$ cm

Volume = area of $\Delta BDC \times$ length

$= \dfrac{1}{2}(8.54)(5.2) \times 18$

$= 399.67 \approx 400$ cm^3

(b) In ΔCED, using Pythagoras theorem,

$EC = \sqrt{ED^2 + DC^2}$

$= \sqrt{18^2 + 8.54^2}$

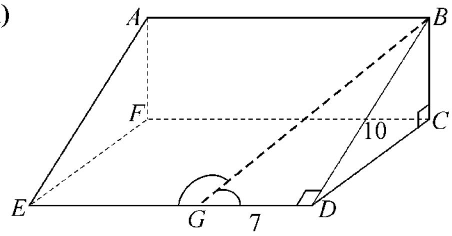
$= \sqrt{396.93} \approx 19.92$ cm

In ΔBEC, $\tan B\hat{E}C = \dfrac{BC}{EC}$

$\Rightarrow \tan B\hat{E}C = \dfrac{5.2}{19.92}$

$\Rightarrow B\hat{E}C \approx 14.6°$

(ii)



In ΔBDG, $B\hat{D}G = 90°$

$\tan B\hat{G}D = \dfrac{10}{7}$

$\Rightarrow B\hat{G}D = \tan^{-1}\left(\dfrac{10}{7}\right) = 55°$

$\therefore B\hat{G}E = 180° - 55° = 125°$

3. **(a)** **(i)** $BC^2 = 8^2 + 13^2 - 2(8)(13)\cos 79°$

$\Rightarrow BC^2 = 193.31$

$\Rightarrow BC = \sqrt{193.31} \Rightarrow BC \approx 13.9$ m

(ii) $\dfrac{\sin A\hat{C}B}{13} = \dfrac{\sin 79°}{13.9}$

$\Rightarrow \sin A\hat{C}B = \dfrac{\sin 79°}{13.9} \times 13$

$\Rightarrow A\hat{C}B = 66.6°$

(b) **(i)** Area of $\Delta DEF = 70 \text{ m}^2$

$\Rightarrow \frac{1}{2}(x+4)(4x-5)\sin 30° = 70$

$\Rightarrow \frac{1}{2}(4x^2-5x+16x-20)\left(\frac{1}{2}\right) = 70$

$\Rightarrow \frac{1}{4}(4x^2+11x-20) = 70$

$\Rightarrow 4x^2+11x-20 = 280$

$\Rightarrow 4x^2+11x-300 = 0$

(ii) $4x^2+11x-300=0$

Using quadratic formula,

$$x = \frac{-11\pm\sqrt{(11)^2-4(4)(-300)}}{2(4)}$$

$$= \frac{-11\pm\sqrt{4921}}{8}$$

$$\Rightarrow x = \frac{-11+70.15}{8} \text{ or } x = \frac{-11-70.15}{8}$$

$$= 7.39 \qquad\qquad = -10.14$$

$\therefore\ x = 7.39, \text{ or } x = -10.14$

(iii) Length of $DE = x+4$

$= 7.39+4$

$= 11.39 \approx 11.4 \text{ m}$

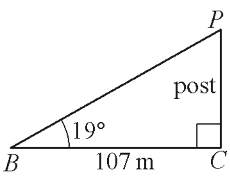
4. **(a)** Let vertical post be CP.

In ΔBCP,

$\tan 19° = \frac{CP}{107}$

$\Rightarrow\ CP = 107\tan 19°$

$\approx 36.8 \text{ m}$



(b) $\cos B\hat{A}C = \frac{158^2+132^2-107^2}{(2)(158)(132)}$

$\Rightarrow\ \cos B\hat{A}C = \frac{30939}{41712}$

$\Rightarrow\ B\hat{A}C = \cos^{-1}\left(\frac{30939}{41712}\right) = 42.1°$

(c) $\frac{\sin C\hat{A}D}{86} = \frac{\sin 116°}{132}$

$\Rightarrow\ \sin C\hat{A}D = \frac{\sin 116°}{132}\times 86$

$\Rightarrow\ \sin C\hat{A}D = 0.5856 \quad \Rightarrow\ C\hat{A}D = 35.8°$

(d) In ΔACD, $\quad A\hat{C}D = 180°-116°-35.8°$

$= 28.2°$

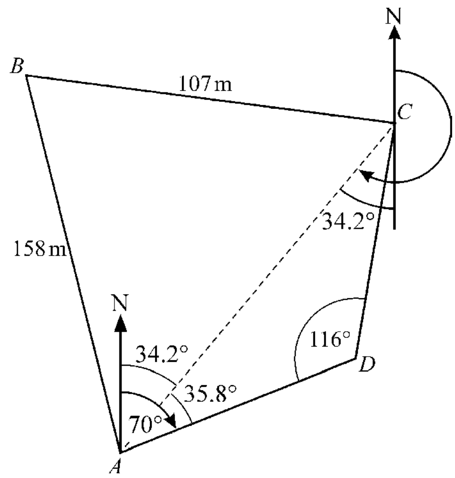
Area of field = area of ΔABC + area of ΔACD

$= \frac{1}{2}(158)(132)\sin 42.1° + \frac{1}{2}(132)(86)\sin 28.2°$

$= 6991.2 + 2682.2$

$= 9673.4 \approx 9670 \text{ m}^2$ (3 sf)

(e)



$C\hat{A}N = 70° - 35.8° = 34.2°$

$\therefore$ Bearing of A from $C = 180° + 34.2°$

$= 214.2°$

5. **(a)** Using cosine rule,

$BC = \sqrt{(115)^2+(80)^2-2(115)(80)(\cos 72°)}$

$= \sqrt{13\,939.087}$

$= 118.06 \approx 118.1 \text{ m}$

(b) Using sine rule,

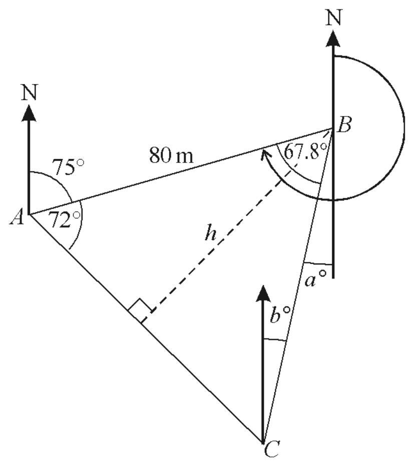
$\frac{\sin A\hat{B}C}{115} = \frac{\sin 72°}{118.1}$

$\Rightarrow\ \sin A\hat{B}C = \frac{\sin 72°}{118.1}\times 115$

$\Rightarrow\ \sin A\hat{B}C = 0.92609$

$\Rightarrow\ A\hat{B}C \approx 67.8°$

(c)



(i) Bearing of A from $B = 180° + 75° = 255°$

(ii) $a° = 75° - 67.8° = 7.2°$

Bearing of B from $C = b°$

$= 007.2°$

(d) Average speed $= \dfrac{\text{distance}}{\text{time}}$

$$= \frac{115}{35}$$

$$= \frac{23}{7} \text{ m/s}$$

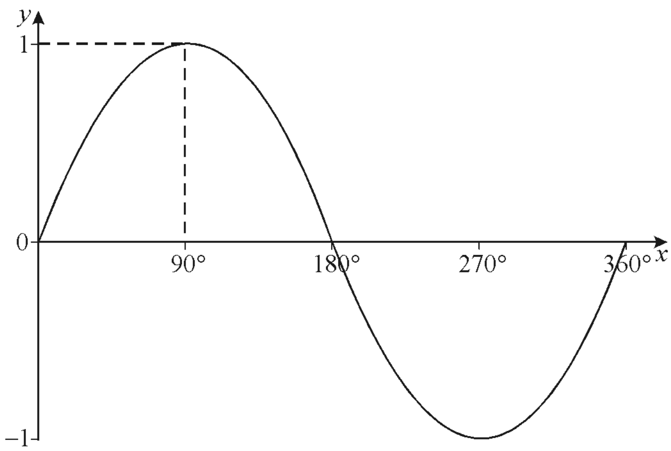
$$= \frac{23}{7} \times \frac{3600}{1000} \approx 11.8 \text{ km/h}$$

(e) From figure above, the shortest distance from B to AC is h

$\therefore \quad \sin 72° = \dfrac{h}{80}$

$\Rightarrow \quad h = 80 \sin 72° \approx 76.1$ m

6. (a) (i)



(ii) Line symmetry = 0
Order of rotational symmetry is 2 about the point (180°, 0°).

(b) $4\sin x - 1 = 2$

$\Rightarrow \quad 4\sin x = 3 \quad \Rightarrow \quad \sin x = \dfrac{3}{4}$

Basic Angle $= \sin^{-1}\left(\dfrac{3}{4}\right) \approx 48.6°$

$\therefore \quad x = 48.6°, \quad 180° - 48.6°$

$\Rightarrow \quad x = 48.6°, \quad 131.4°$

7. (a) In ΔPQR, using cosine rule,

$QR = \sqrt{120^2 + 150^2 - 2(120(150)\cos 25°}$

$= \sqrt{4272.92} \approx 65.4$ m

(b) $P\hat{S}R = 180° - 55° - 45° = 80°$

In ΔPSR, using sine rule,

$$\frac{RS}{\sin 55°} = \frac{150}{\sin 80°}$$

$$\Rightarrow \quad RS = \frac{150}{\sin 80°} \times \sin 55°$$

$$= 124.768 \approx 125 \text{ m}$$

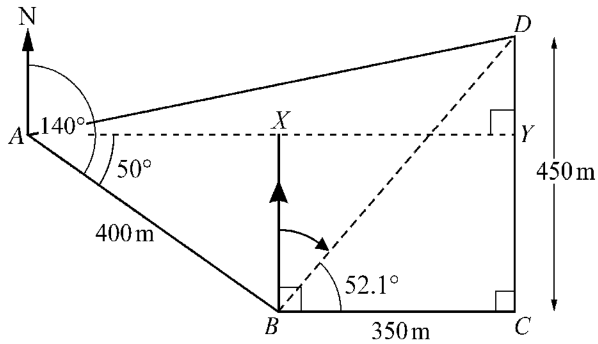
(c) Total area = Area of ΔPQR + Area of ΔPSR

$$= \frac{1}{2}(150)(120)\sin 25° + \frac{1}{2}(150)(124.768)\sin 45°$$

$$= 3803.56 + 6616.82$$

$$= 10420.38 \approx 10400 \text{ m}^2 \text{ (to 3s.f)}$$

8. (a)



In $\Delta BCD, \quad \tan C\hat{B}D = \dfrac{450}{350}$

$\Rightarrow \quad C\hat{B}D = \tan^{-1}\left(\dfrac{450}{350}\right) = 52.1°$

$\therefore \quad$ Bearing of D from $B = 90° - 52.1°$

$= 037.9°$

(b) From figure above, in ΔAXB,

$B\hat{A}X = 140° - 90° = 50°$,

$\sin 50° = \dfrac{BX}{400} \Rightarrow BX = 400\sin 50° \approx 306.42$ m

$\therefore \quad CY = BX = 306.42$ m

$\cos 50° = \dfrac{AX}{400} \Rightarrow AX = 400\cos 50° \approx 257.1$ m

Now, $AY = AX + XY$

$= 257.1 + 350 = 607.1$ m

and $YD = CD - CY$

$= 450 - 306.42 = 143.58$ m

In ΔAYD, using Pythagoras theorem,

$AD = \sqrt{AY^2 + YD^2}$

$\Rightarrow \quad AD = \sqrt{(607.1)^2 + (143.58)^2}$

$= \sqrt{389185.63} = 623.85 \approx 624$ m

(c) Total distance covered $= 400 + 350 + 450 + 624$

$= 1824$ m

Total time $= \dfrac{\text{total distance}}{\text{speed}}$

$= \dfrac{1824}{3} = 608$ seconds

$= 10$ minutes 8 seconds